

No Classicalization Beyond Spherical Symmetry

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(Dated: September 20, 2011)

We point out that a field theory that exhibits the classicalization phenomenon for perfect spherical symmetry ceases to do so when the spherical symmetry is significantly relaxed. We first investigate a small non-spherical deformation and show that the classicalization radius tends to decrease in a region where a shell made of the field is slightly flattened. Next, in order to describe a sufficiently large flattened region, we consider a high-energy collision of planar shells and show that the system never classicalizes before reaching sub-cutoff lengths. This no-go result is further strengthened by an analysis of a small non-planar deformation. Finally, we show that the shape of a scattered planar wave is UV sensitive.

I. INTRODUCTION

Recently a novel approach to the UV-completion of a class of non-renormalizable theories was proposed [1]. In this approach, called *classicalization*, high-energy scattering amplitudes are conjectured to be unitarized by the production of extended classical objects, dubbed *classicalons*. Although the existence and properties of classicalons have not been studied at all, it has been somehow suspected that they might form below the length scale called the *classicalization radius* [2, 4]. The classicalization radius is defined as the classical radius down to which high-energy shells of fields propagate essentially freely, without experiencing a significant correction from interaction. We can at least consider the condition

$$(\text{classicalization radius}) \gg (\text{cutoff length}) \quad (1)$$

as a necessary condition for the formation of classicalons.

If classicalization really works then we would have to change our view of non-renormalizable field theories and Wilsonian UV completion, in which a weakly coupled quantum field theory above the cutoff scale of an effective field theory is reconstructed by integrating-in some new degrees of freedom. In the classicalization approach, a non-renormalizable field theory would unitarize itself by its own resources, above the scale that would otherwise be the UV cutoff of the theory.

Hence it is important to investigate the validity of the classicalization proposal. However, so far, the studies of the classicalization approach have been rather limited. One of the limitations is that all considerations in the literature assume perfect spherical symmetry. The purpose of this paper is to study the classicalization proposal beyond spherical symmetry. Unfortunately, we shall see that the above mentioned necessary condition (1) does

not hold if spherical symmetry is significantly relaxed. Thus, considerations in the present paper point towards the conclusion that classicalization does not serve as UV-completion for the class of non-renormalizable theories.

The rest of the paper is organized as follows. In Sec. II we shall briefly review the classicalization proposal. We shall also argue that classicalization does not serve as a way to UV complete the other classes of the field theories considered in the literature, except for the Goldstone-type field theory. For this reason, we shall investigate the Goldstone-type field theory in starting with Sec. III. In Sec. III we consider a small non-spherical deformation and show that the classicalization radius tends to decrease in a region where a shell made of the field is slightly flattened. In Sec. IV, in order to describe a sufficiently large flattened region, we consider the high-energy collision of planar shells and show that the system never classicalizes before reaching sub-cutoff lengths. This no-go result is further strengthened by the analysis of a small non-planar deformation in Sec. V. In Sec. VI we show that the shape of scattered planar wave is UV sensitive. Sec. VII is devoted to a summary of the main results of this paper.

Throughout this paper we shall adopt the mostly plus sign $(- + + +)$ for the metric.

II. A BRIEF REVIEW OF THE CLASSICALIZATION PROPOSAL

In the literature, the classicalization proposal has been studied for a scalar field without shift symmetry, a scalar field with shift symmetry (Goldstone-type field), and the graviton in general relativity. In this paper we shall investigate the robustness of the classicalization proposal for the theory of Goldstone bosons. Before going into further detail, let us explain why we will not consider the other two cases.

The example of the scalar field without shift symmetry

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has a Lagrangian of the form

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{\phi}{2M_*}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \dots \quad (2)$$

In ref. [1], the second term on the right hand side was considered as a possible source of classicalization. However, one can redefine the field ϕ so that the sum of the first and the second terms becomes a canonical kinetic term of the new field $\phi_c = (2/3)(1 + \phi/M_*)^{3/2}$. Thus, the system is equivalent to a noninteracting canonical scalar field. The field redefinition does not work for $\phi < -M_*$, but in this regime $\bar{\phi}_c = |\phi_c|$ has a wrong sign kinetic term and unitarity is violated. Therefore, the first two terms in this Lagrangian do not serve as a good example of classicalization.

The graviton in Einstein gravity has been considered as a possible candidate that exhibits the classicalization phenomenon [4, 5]. However, it is well known that in general relativity, one can form a black hole with arbitrarily small mass [8]. Therefore, in general relativity one can in principle probe arbitrarily short distances unless new physics above the cutoff scale somehow prevents one from doing so. Therefore, classicalization is unlikely to work in general setups in general relativity.

In this paper we shall consider the Goldstone-type field, i.e. a scalar field with shift symmetry, since this is the only remaining situation that has been studied in the literature so far.

Classicalization of the Goldstone-type field was investigated in the Euclidean path integral formulation in [6]. However, the background solution is singular and the corresponding Euclidean action is infinite. Therefore, it seems rather difficult to extract any physically meaningful insights from the result of [6]. For this reason, in the present paper we will not adopt the path integral formulation.

The Goldstone-type field that has been studied in the context of classicalization has a Lagrangian of the form

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{L_*^4}{4}[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi]^2. \quad (3)$$

Following refs. [2, 4], we investigate the classical dynamics of a spherical shell made of this scalar field in a Minkowski spacetime. Namely, we assume that the back-reaction of the scalar field on the background geometry is negligible and we seek a solution $\phi(t, r)$ to the equation of motion

$$\square_4\phi = L_*^4\partial^\mu[(\partial\phi)^2\partial_\mu\phi] \quad (4)$$

in a Minkowski spacetime

$$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2(\theta)d\bar{\theta}^2). \quad (5)$$

Our neglect of the backreaction effects is justified in the decoupling limit $M_{pl}L_* \gg 1$.

What we are interested in is the classical radius r_* down to which high-energy spherical shells propagate essentially freely, without experiencing a significant correction from the interaction term. This radius is dubbed

the *classicalization radius*. In order to determine r_* for this system, refs. [2, 4] solved the equation of motion iteratively by expanding ϕ as

$$\phi = \phi_0 + \phi_1 + \dots, \quad (6)$$

and assuming that ϕ_0 satisfies the equation of motion with $L_* = 0$. We shall soon see what the expansion parameter is.

Under the assumed spherical symmetry, a general solution to the zeroth order equation $\square_4\phi_0 = 0$ is

$$\phi_0 = \frac{1}{r}[F_0(t+r) - F_0(t-r)], \quad (7)$$

where we have imposed the regularity of ϕ_0 at $r = 0$. In order for this solution to represent a shell with thickness a , the form of the function $F_0(t)$ is set to

$$F_0(t) = Af(t/a), \quad (8)$$

where $f(\tau)$ is a function whose amplitude and derivatives are of $O(1)$ in the vicinity of the shell configuration, so that the total energy and the occupation number of the configuration are $\sim A^2/a$ and $\sim A^2$, respectively. According to refs. [2, 4], we are interested in configurations with a small occupation number. Thus we suppose that

$$A = O(1). \quad (9)$$

We are interested in the behavior of the system before the thin shell peaked at $t+r \sim 0$ reaches the center. Hence we can safely drop $-F_0(t-r)$. We thus have

$$\phi_0 = \frac{A}{r}f(\tau_+), \quad \tau_+ = \frac{t+r}{a}. \quad (10)$$

Since ϕ_0 is small at large r but becomes large at small r , the nonlinear interaction is negligible at large r but becomes significant at small r . The classicalization radius r_* for this system is thus defined as a radius at which the amplitude of ϕ_1 catches up with that of ϕ_0 .

One can easily guess r_* from (10). Since

$$g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = -\frac{A^2}{ar^3}[f'(\tau_+)f(\tau_+) + O(a/r)], \quad (11)$$

the second term in the Lagrangian (3) becomes as important as the first term when

$$\frac{L_*^4}{ar^3} \sim 1. \quad (12)$$

Thus, it is expected that the classicalization radius r_* is

$$r_* \sim (L_*^4/a)^{1/3}. \quad (13)$$

To confirm this expectation, let us seek the first order solution ϕ_1 , which is the retarded solution to

$$\square_4\phi_1 = L_*^4\partial^\mu[(\partial\phi_0)^2\partial_\mu\phi_0]. \quad (14)$$

Since the right hand side is calculated as

$$\bar{s}_0(\tau_+, r) = \frac{A^3 L_*^4}{2a^2 r^5} \left\{ [f''(\tau_+) (f(\tau_+))^2 + 4(f'(\tau_+))^2 f(\tau_+)] + O(a/r) \right\}, \quad (15)$$

we obtain

$$\begin{aligned} \phi_1 &= \frac{a}{2r} \int_r^\infty dr' r' \int_{-\infty}^{\tau_+} d\tau_1 \bar{s}_0(\tau_1, r') \\ &= -\frac{A^3 L_*^4}{12a^4 r^4} \int_{-\infty}^{\tau_+} d\tau_1 \left\{ [f''(\tau_1) (f(\tau_1))^2 + 4(f'(\tau_1))^2 f(\tau_1)] + O(a/r) \right\}. \end{aligned} \quad (16)$$

Since the integral is of $O(1)$ in the vicinity of the shell, the amplitude of ϕ_1 catches up with that of ϕ_0 at $r \sim r_*$, where r_* is given by (13).

III. NON-SPHERICAL DEFORMATION

In this section we consider a non-spherical deformation of the shell and see that if a part of the shell is flatter (or more curved), r_* is slightly reduced (or increased) in that part.

A. zeroth order solution

For the zeroth order part, we adopt the ansatz

$$\phi_0 = \phi_{00}(t, r) + \phi_{02}(t, r) P_2(\cos \theta). \quad (17)$$

Then the zeroth order equation leads to the solution

$$\begin{aligned} \phi_{00} &= \frac{1}{r} [F_0(t+r) + G_0(t-r)], \\ \phi_{02} &= r \left(\partial_r \frac{1}{r} \right)^2 [F_2(t+r) + G_2(t-r)]. \end{aligned} \quad (18)$$

Imposing regularity at $r = 0$ leads to the condition $F_0(t) + G_0(t) = F_2(t) + G_2(t) = 0$ for $\forall t$. Thus, the zeroth order solution is

$$\begin{aligned} \phi_0 &= \frac{1}{r} [F_0(t+r) - F_0(t-r)] \\ &+ r \left(\partial_r \frac{1}{r} \right)^2 [F_2(t+r) - F_2(t-r)] P_2(\cos \theta) \end{aligned} \quad (19)$$

Let us suppose that

$$F_0(t) = A f(t/a), \quad F_2(t) = A a r_2 \int_{-\infty}^{t/a} d\tau_1 f(\tau_1), \quad (20)$$

where r_2 is a constant. For $r \gg a$, this solution is reduced to

$$\begin{aligned} \phi_0 &\simeq \frac{A}{r} \left\{ [f(\tau_+) - f(\tau_-)] + [f'(\tau_+) - f'(\tau_-)] \frac{r_2}{a} P_2(\cos \theta) \right\} \\ &= \frac{A}{r} [f(\tilde{\tau}_+) - f(\tilde{\tau}_-)] + O(r_2^2/a^2), \end{aligned} \quad (21)$$

where

$$\tau_{\pm} \equiv \frac{t \pm r}{a}, \quad \tilde{\tau}_{\pm} = \tau_{\pm} + \frac{r_2}{a} P_2(\cos \theta). \quad (22)$$

If $f(\tau)$ represents a wave packet peaked at $\tau = 0$ then $f(\tilde{\tau}_{\pm})$ is peaked at $r \simeq \mp[t + r_2 P(\cos \theta)]$.

The non-spherical deformation ϕ_{02} includes $O(a/r)$ and $O(a^2/r^2)$ corrections. The $O(a/r)$ correction to (21) slightly changes the height of the peak. Indeed, by including the $O(a/r)$ correction, (21) is modified as

$$\begin{aligned} \phi_0 &= \frac{A}{r} \left\{ f(\tilde{\tau}_+) \left[1 - \frac{3r_2}{r} P_2(\cos \theta) \right] \right. \\ &\quad \left. - f(\tilde{\tau}_-) \left[1 + \frac{3r_2}{r} P_2(\cos \theta) \right] \right. \\ &\quad \left. + O(a^2/r^2) + O(r_2^2/a^2) \right\}, \end{aligned} \quad (23)$$

Although we shall not write it explicitly here, the $O(a^2/r^2)$ correction represents the tail induced by the motion of the deformed shell.

B. Retarded solution to inhomogeneous equation

It is easy to show that

$$\phi_1(t_+, t_-) = \frac{\psi_{10}}{r} + r \left(\partial_r \frac{1}{r} \right)^2 \psi_{12} P_2(\cos \theta) \quad (24)$$

with

$$\psi_{1n} = -\frac{1}{4} \int_{-\infty}^{t_+} dt'_+ \int_{-\infty}^{t_-} dt'_- f_n(t'_+, t'_-) \quad (n = 0, 2) \quad (25)$$

satisfies the inhomogeneous equation

$$\square_4 \phi_1 = \frac{f_0(t_+, t_-)}{r} + r \left(\partial_r \frac{1}{r} \right)^2 f_2(t_+, t_-) P_2(\cos \theta) \quad (26)$$

and the retarded boundary condition. Note $t_{\pm} \equiv t \pm r$.

There is freedom to add $B_2(t)r^3 + C_2(t)r$ to $f_2(t_+, t_-)$ but $B_2(t)$ and $C_2(t)$ can be fixed by demanding that $f_2(t_+, t_-)$ remains finite at $r \rightarrow \infty$ for $\forall t$. Thus, when the inhomogeneous equation is of the form

$$\square_4 \phi_1 = s_0(t, r) + s_2(t, r) P_2(\cos \theta), \quad (27)$$

we have

$$\begin{aligned} f_0(t_+, t_-) &= r s_0(t, r), \\ f_2(t_+, t_-) &= r \int_r^\infty dr_1 r_1 \int_{r_1}^\infty dr_2 \frac{s_2(t, r_2)}{r_2}, \end{aligned} \quad (28)$$

where t and r on the right hand side should be understood as $(t_+ + t_-)/2$ and $(t_+ - t_-)/2$ respectively.

C. First order solution up to $O(r_2/a)$ with $O(a/r)$ correction

We are interested in the behavior of the system before the thin shell peaked at $t + r \sim 0$ reaches the center. Hence, we can safely drop $-F_0(t - r)$ and $-F_2(t - r)$ in (19). We thus have

$$\begin{aligned}\phi_0 &= \frac{A}{r} \left\{ f(\tau_+) \left[1 - \frac{3r_2}{r} P_2(\cos \theta) \right] \right. \\ &\quad \left. + f'(\tau_+) \frac{r_2}{a} P_2(\cos \theta) + O(a^2/r^2) \right\} \\ &= \frac{A}{r} \left\{ f(\tilde{\tau}_+) \left[1 - \frac{3r_2}{r} P_2(\cos \theta) \right] \right. \\ &\quad \left. + O(a^2/r^2) + O(r_2^2/a^2) \right\},\end{aligned}\quad (29)$$

where τ_+ and $\tilde{\tau}_+$ are defined in (22). Consequently, up to $O(r_2/a)$ the equation for ϕ_1 is of the form (27) with

$$\begin{aligned}\bar{s}_0 &= \frac{A^3 L_*^4}{2a^2 r^5} \left\{ [f''(\tau_+)(f(\tau_+))^2 + 4(f'(\tau_+))^2 f(\tau_+)] \right. \\ &\quad \left. - \frac{3l}{r} f'(\tau_+)(f(\tau_+))^2 + O(a^2/r^2) \right\}, \\ \bar{s}_2 &= \frac{r_2}{a} \left\{ \frac{\partial \bar{s}_0}{\partial \tau_+} \right. \\ &\quad \left. - \frac{15A^3 L_*^4}{2ar^6} [f''(\tau_+)(f(\tau_+))^2 + 4(f'(\tau_+))^2 f(\tau_+)] \right. \\ &\quad \left. + \frac{A^3 L_*^4}{a^2 r^5} \times O(a^2/r^2) \right\}.\end{aligned}\quad (30)$$

where $\bar{s}_n \equiv s_n(t = a\tau_+ - r, r)$ ($n = 0, 2$) is s_n written as a function of (τ_+, r) . Note that in the coordinates $(\tau_+, r, \theta, \bar{\theta})$, the Minkowski metric (5) is written as

$$ds^2 = -a^2 d\tau_+^2 + 2ad\tau_+ dr + r^2(d\theta^2 + \sin^2 \theta d\bar{\theta}^2), \quad (31)$$

and

$$g^{\mu\nu} \partial_\mu \partial_\nu = \frac{1}{a} \partial_+ \partial_r + \frac{1}{4} \partial_r^2 + \frac{1}{r^2} \partial_\theta^2 + \frac{1}{r^2 \sin^2 \theta} \partial_{\bar{\theta}}^2. \quad (32)$$

From \bar{s}_2 , f_2 is obtained by solving

$$\begin{aligned}\bar{s}_2 &= r \left[\left(\frac{\partial}{\partial r} + \frac{1}{a} \frac{\partial}{\partial \tau_+} \right) \frac{1}{r} \right]^2 \bar{f}_2 \\ &= \frac{1}{a^2 r} \left[\frac{\partial^2}{\partial \tau_+^2} - \frac{3a}{r} \frac{\partial}{\partial \tau_+} + O(a^2/r^2) \right] \bar{f}_2.\end{aligned}\quad (33)$$

Here and in the following, $\bar{f}_n \equiv f_n(t_+ = a\tau_+, t_- = a\tau_+ - 2r)$ ($n = 0, 2$) is f_n written as a function of (τ_+, r) . Thus, we obtain

$$\begin{aligned}\bar{f}_2(\tau_+, r) &= a^2 r \left[\int_{-\infty}^{\tau_+} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \bar{s}_2(\tau_2, r) \right. \\ &\quad \left. + \frac{3a}{r} \int_{-\infty}^{\tau_+} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \int_{-\infty}^{\tau_2} d\tau_3 \bar{s}_2(\tau_3, r) \right. \\ &\quad \left. + O(a^2/r^2) \right].\end{aligned}\quad (34)$$

We of course have

$$\bar{f}_0(\tau_+, r) = r \bar{s}_0(r, \tau_+). \quad (35)$$

Now by changing the integration variables from (τ_+', t_-') to $(\tau'_+ = t'_+/a, r' = (t'_+ - t'_-)/2)$, the formula (25) is written as

$$\bar{\psi}_{1n}(\tau_+, r) = -\frac{a}{2} \int_r^\infty dr' \int_{-\infty}^{\tau_+} d\tau'_+ \bar{f}_n(\tau'_+, r') \quad (n = 0, 2), \quad (36)$$

where $\bar{\psi}_{1n} \equiv \psi_{1n}(t_+ = a\tau_+, t_- = a\tau_+ - 2r)$ is ψ_{1n} written as a function of (τ_+, r) . Concretely,

$$\begin{aligned}\bar{\psi}_{10}(\tau_+, r) &= -\frac{a}{2} \int_r^\infty dr' r' \int_{-\infty}^{\tau_+} d\tau_1 \bar{s}_0(\tau_1, r'), \\ \bar{\psi}_{12}(\tau_+, r) &= -\frac{a^3}{2} \int_r^\infty dr' r' \\ &\quad \times \left[\int_{-\infty}^{\tau_+} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \int_{-\infty}^{\tau_2} d\tau_3 \bar{s}_2(\tau_3, r') \right. \\ &\quad + \frac{3a}{r'} \int_{-\infty}^{\tau_+} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \int_{-\infty}^{\tau_2} d\tau_3 \\ &\quad \left. \times \int_{-\infty}^{\tau_3} d\tau_4 \bar{s}_2(\tau_4, r') + O(a^2/r'^2) \right].\end{aligned}\quad (37)$$

Therefore, up to $O(r_2/a)$, the formula (24) gives

$$\phi_1 \simeq \bar{\phi}_{10}(\tau_+, r) + \bar{\phi}_{12}(\tau_+, r) P_2(\cos \theta), \quad (38)$$

where

$$\begin{aligned}\bar{\phi}_{10} &= -\frac{a}{2r} \int_r^\infty dr' r' \int_{-\infty}^{\tau_+} d\tau_1 \bar{s}_0(\tau_1, r'), \\ \bar{\phi}_{12} &= -\frac{a}{2r} \left[\int_r^\infty dr' r' \int_{-\infty}^{\tau_+} d\tau_1 \bar{s}_2(\tau_1, r') \right. \\ &\quad \left. - \frac{3a}{r} \int_r^\infty dr' \int_{r'}^\infty dr'' \int_{-\infty}^{\tau_+} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \bar{s}_2(\tau_2, r') \right. \\ &\quad \left. + O(a^2/r^2) \right].\end{aligned}\quad (39)$$

Here, we have integrated by parts in order to obtain the expression for $\bar{\phi}_{12}$. Finally, by using the explicit expressions (30), it is shown that

$$\begin{aligned}\bar{\phi}_{10} &= -\frac{A^3 L_*^4}{12ar^4} \left\{ g(\tau_+) - \frac{3a}{4r} (f(\tau_+))^3 + O(a^2/r^2) \right\}, \\ \bar{\phi}_{12} &= -\frac{A^3 L_*^4}{12ar^4} \frac{r_2}{a} \left\{ g'(\tau_+) - \frac{9a}{4r} f'(\tau_+) (f(\tau_+))^2 \right. \\ &\quad \left. - \frac{12a}{r} g(\tau_+) \right\},\end{aligned}\quad (40)$$

where

$$g(\tau_+) \equiv \int_{-\infty}^{\tau_+} d\tau_1 [f''(\tau_1)(f(\tau_1))^2 + 4(f'(\tau_1))^2 f(\tau_1)]. \quad (41)$$

In summary,

$$\begin{aligned}\phi_1 = & -\frac{A^3 L_*^4}{12a r^4} \left\{ g(\tilde{\tau}_+) \left[1 - \frac{12r_2}{r} P_2(\cos \theta) \right] - \frac{3a}{4r} (f(\tilde{\tau}_+))^3 \right. \\ & \left. + O(a^2/r^2) + O(r_2^2/a^2) \right\}.\end{aligned}\quad (42)$$

Hence, we obtain

$$\begin{aligned}\frac{\phi_1}{\phi_0} = & -\frac{A^2 L_*^4}{12a r^3} \left\{ \frac{g(\tilde{\tau}_+)}{f(\tilde{\tau}_+)} \left[1 - \frac{9r_2}{r} P_2(\cos \theta) \right] - \frac{3a}{4r} (f(\tilde{\tau}_+))^2 \right. \\ & \left. + O(a^2/r^2) + O(r_2^2/a^2) \right\}.\end{aligned}\quad (43)$$

As noted earlier, the classicalization radius r_* is the value of r at which the amplitude of ϕ_1 catches up with that of ϕ_0 . Thus, this result clearly shows that r_* is slightly reduced (or increased) if $r_2 P_2(\cos \theta)$ is positive (or negative), i.e. if the shell is flatter (or more curved) in some region. This is exactly what we have expected.

In the next section, in order to describe a sufficiently large flattened region, we consider a planarly symmetric case.

IV. PLANARLY SYMMETRIC CASE

In 4-dimensional flat spacetime

$$ds_4^2 = -dt^2 + dx^2 + dy^2 + dz^2, \quad (44)$$

we consider a planarly symmetric ansatz $\phi = \phi(t, x)$. By integrating out the two spatial dimensions parallel to the plane of symmetry, y and z , we obtain a scalar field $\varphi = \sqrt{\mathcal{A}_2} \phi$ in 2-dimension, where \mathcal{A}_2 is the area of the plane.

In a two dimensional flat spacetime

$$ds_2^2 = -dt^2 + dx^2 = -dt_+ dt_-, \quad t_\pm \equiv t \pm x, \quad (45)$$

the retarded solution to

$$\square_2 \varphi_1 = f(t_+, t_-) \quad (46)$$

is given by

$$\varphi_1 = -\frac{1}{4} \int_{-\infty}^{t_+} dt'_+ \int_{-\infty}^{t_-} dt'_- f(t'_+, t'_-). \quad (47)$$

The equation of motion

$$\square_4 \phi = L_*^4 \partial^\mu [(\partial\phi)^2 \partial_\mu \phi] \quad (48)$$

in 4-dimensions is reduced to

$$\square_2 \varphi = \frac{L_*^4}{\mathcal{A}_2} \partial^\mu [(\partial\varphi)^2 \partial_\mu \varphi] \quad (49)$$

in 2-dimensions. Let us solve this equation iteratively by expanding φ as

$$\varphi = \varphi_0 + \varphi_1 + \dots, \quad (50)$$

and assuming that φ_0 satisfies the equation of motion with $L_* = 0$, i.e., $\square_2 \varphi = 0$. Thus,

$$\varphi_0 = \varphi_+(t_+) + \varphi_-(t_-). \quad (51)$$

We shall soon see what the expansion parameter is.

The first order equation is

$$\square_2 \varphi_1 = \frac{8L_*^4}{\mathcal{A}_2} [\varphi_+''(t_+) (\varphi_-'(t_-))^2 + \varphi_-(t_-)'' (\varphi_+'(t_+))^2]. \quad (52)$$

Thus the retarded solution is

$$\begin{aligned}\varphi_1 = & -\frac{2L_*^4}{\mathcal{A}_2} \int_{-\infty}^{t_+} dt'_+ \int_{-\infty}^{t_-} dt'_- [\varphi_+''(t'_+) (\varphi_-'(t'_-))^2 \\ & + \varphi_-(t'_-)' (\varphi_+'(t'_+))^2].\end{aligned}\quad (53)$$

Let us suppose that

$$\varphi_+(t) = \varphi_-(t) = A f(t/a). \quad (54)$$

Roughly speaking, the total energy and the occupation number of the configuration are $\sim A^2/a$ and $\sim A^2$. Then we obtain

$$\begin{aligned}\frac{\varphi_1}{A} = & -\frac{2L_*^4 A^2}{\mathcal{A}_2 a^2} \int_{-\infty}^{t_+/a} d\tau_1 \int_{-\infty}^{t_-/a} d\tau_2 [f''(\tau_1) (f'(\tau_2))^2 \\ & + f''(\tau_2) (f'(\tau_1))^2].\end{aligned}\quad (55)$$

We now see that the expansion parameter is

$$\epsilon \equiv \frac{2L_*^4 A^2}{\mathcal{A}_2 a^2}. \quad (56)$$

The iterative solution is a good approximation if $\epsilon \ll 1$.

As an example, let us consider a Gaussian wave packet

$$f(\tau) = e^{-\tau^2}. \quad (57)$$

Figs 1 and 2 show φ_0/A and $\varphi_1/(-A\epsilon)$, respectively, as functions of $\tau_\pm \equiv t_\pm/a$. This clearly shows that φ_1 stays small compared to φ_0 as far as $\epsilon \ll 1$ and that the interaction length is $\sim a$. In this case the system never classicalizes. For $\epsilon \sim 1$, the amplitude of φ_1 becomes as large as that of φ_0 but the interaction length is still $\sim a$. Thus, for high-energy scattering with $a \ll L_*$, there is no sign of classicalization before reaching the sub-cutoff length.

V. DEVIATION FROM PERFECT PLANAR SYMMETRY

In this section we will consider deviations from the perfect planar symmetry discussed above. Our purpose in doing so is to see if the fluctuations will introduce a classicalization radius. Since classicalization was shown for spherical symmetry, our expectation is to observe a tendency away from the results of the strictly planar symmetric case.

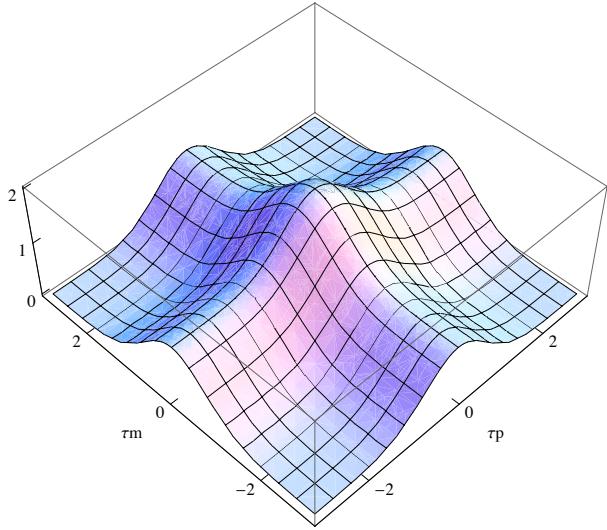


FIG. 1: The plot of φ_0/A as a function of $\tau_{\pm} \equiv t_{\pm}/a$ for $f(\tau) = e^{-\tau^2}$.

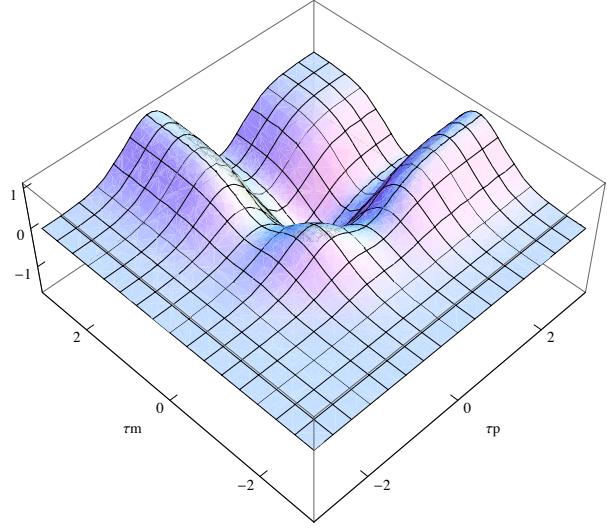


FIG. 2: The plot of $\varphi_1/(-A\epsilon)$ as a function of $\tau_{\pm} \equiv t_{\pm}/a$ for $f(\tau) = e^{-\tau^2}$.

We want to probe in a 4-dimensional spacetime, the behavior of the field:

$$\phi(t, x, y, z) = \phi_0(t, x, y, z) + \phi_1(t, x, y, z) + \dots, \quad (58)$$

where ϕ_0 satisfies $\square_4 \phi_0 = 0$. In terms of the rescaled fields φ of the previous section, let us make the ansatz:

$$\begin{aligned} \phi_0(t, x, y, z) &= \frac{1}{\sqrt{\mathcal{A}_2}} \varphi_{00}(t, x) \\ &+ \frac{1}{\sqrt{\mathcal{A}_2}} \varphi_{01}(t, x) \operatorname{Re}(e^{i(k_y y + k_z z)}). \end{aligned} \quad (59)$$

In this case, in order for $\square_4 \phi_0 = 0$ to be satisfied, we must have:

$$\begin{aligned} \square_2 \varphi_{00} &= 0 \\ (\square_2 - m^2) \varphi_{01} &= 0, \end{aligned} \quad (60)$$

where $m^2 = k_y^2 + k_z^2$. Let us expand φ_{01} in orders of $m^2 a^2$ so that:

$$\varphi_{01} = \varphi_{01}^{(0)} + \varphi_{01}^{(1)} + \dots. \quad (61)$$

Matching powers of $m^2 a^2$ we see that if $\varphi_{01}^{(0)}$ satisfies $\square_2 \varphi_{01}^{(0)} = 0$, then the equation that is to be satisfied by $\varphi_{01}^{(1)}$ is:

$$a^2 \square_2 \varphi_{01}^{(1)} = m^2 a^2 \varphi_{01}^{(0)}. \quad (62)$$

Anticipating that the leading modification, as in section III, due to the deformation will be a shift in the peak position of the wavepacket, we will make the ansatz up to terms of order r/a :

$$\begin{aligned} \varphi_{00} &= A f(\tau_+) + A f(\tau_-) \\ \varphi_{01}^{(0)} &= -\frac{A r_1}{a} [f'(\tau_+) + f'(\tau_-)], \end{aligned} \quad (63)$$

$$\text{where } \tau_{\pm} \equiv \frac{t_{\pm}}{a} \equiv \frac{t \pm x}{a}.$$

We begin our analysis to determine if there is a change in the classicalization radius, r_* by calculating the deformed ϕ_1 to order r_1/a . Then we have for ϕ_0 in this approximation:

$$\begin{aligned} \phi_0(\tau_+, \tau_-, y, z) &\approx \frac{A}{\sqrt{\mathcal{A}_2}} [f(\bar{\tau}_+) + f(\bar{\tau}_-)] \\ &\equiv \frac{A}{\sqrt{\mathcal{A}_2}} f(\bar{\tau}_+, \bar{\tau}_-), \end{aligned} \quad (64)$$

where $\bar{\tau}_{\pm} = \frac{t_{\pm} - r_1 \cos(k_{\perp} \cdot x_{\perp})}{a}$. Note that $f(\bar{\tau}_+)$ is a wave peaked at $x = -t + r_1 \cos(k_{\perp} \cdot x_{\perp})$ and $f(\bar{\tau}_-)$ is a wave peaked at $x = t - r_1 \cos(k_{\perp} \cdot x_{\perp})$. A cartoon of ϕ_0 at a time slice $t < 0$ is shown in figure 3. We are interested in the region where the two plane waves are bent away from each other, which corresponds to a deformation that brings us from the perfect planar case closer to that of the spherical case. For our ansatz, this corresponds to the region where $r_1 \cos(k_{\perp} \cdot x_{\perp})$ is positive. We expect that r_* will tend to increase in this region as we increase r_1 .

We now need to find ϕ_1 using the equation of motion:

$$\square_4 \phi = L_*^4 \partial^{\mu} (\partial_{\mu} \phi (\partial_{\nu} \phi)^2), \quad (65)$$

for which the first order correction is:

$$\square_4 \phi_1 = L_*^4 \partial^{\mu} (\partial_{\mu} \phi_0 (\partial_{\nu} \phi_0)^2). \quad (66)$$

One such contribution to this order, ϕ_{10} , is obtained by substituting $\frac{1}{\sqrt{\mathcal{A}_2}} \varphi_{00}$ for ϕ_0 above.:

$$\square_4 \phi_{10} = \frac{8 L_*^4 A^3}{a^4 \mathcal{A}_2^{3/2}} b(\tau_+, \tau_-), \quad (67)$$

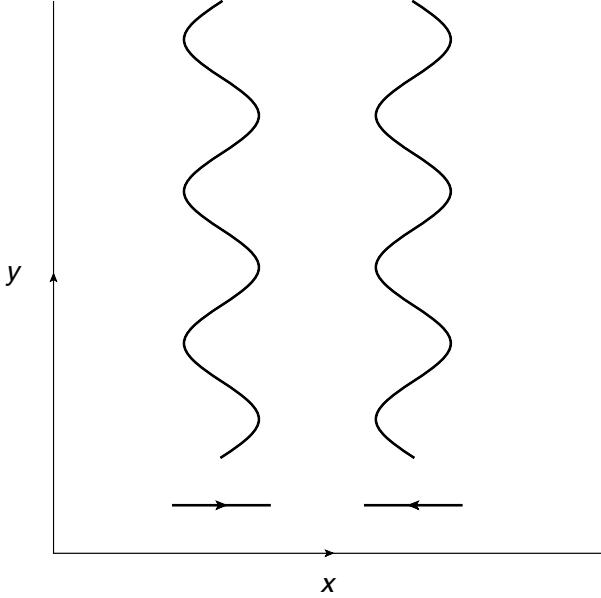


FIG. 3: A sketch showing the peak of ϕ_0 in the x - y plane for some time $t < 0$.

where we have defined $b(\tau_+, \tau_-) = f''(\tau_+)f'(\tau_-)^2 + f''(\tau_-)f'(\tau_+)^2$ for later convenience. It is not hard to show that the solution to the above differential equations in the limit $m^2 a^2 \rightarrow 0$ is:

$$\phi_{10} = -\frac{2L_*^4 A^3}{a^2 \mathcal{A}_2^{3/2}} [g(\tau_+, \tau_-) + \mathcal{O}(m^2 a^2)], \quad (68)$$

where the function $g(\tau_+, \tau_-)$ is given by:

$$g(\tau_+, \tau_-) = \int_{-\infty}^{\tau_+} d\tau'_+ \int_{-\infty}^{\tau_-} d\tau'_- b(\tau'_+, \tau'_-). \quad (69)$$

This solution is just the one presented in the previous section. In the following, just as in the case for ϕ_0 , we will find the contribution to ϕ_1 up to order $\frac{r_1}{a}$. We will denote this contribution $\phi_{11}^{(0)}$.

$\phi_{11}^{(0)}$ can be found by considering the RHS of (66) with linear contributions from $\frac{1}{\sqrt{\mathcal{A}_2}} \varphi_{01}^{(0)} \cos(k_\perp \cdot x_\perp)$ and quadratic contributions from $\frac{1}{\sqrt{\mathcal{A}_2}} \varphi_{00}$. In this case:

$$\square_4 \phi_{11}^{(0)} = -\frac{8L_*^4 A^3 r_1}{a^5 \mathcal{A}_2^{3/2}} \cos(k_\perp \cdot x_\perp) [\partial_{\tau_+} b(\tau_+, \tau_-) + \partial_{\tau_-} b(\tau_+, \tau_-)]. \quad (70)$$

The solution to the above differential equation in the limit $m^2 a^2 \rightarrow 0$ is easily obtained:

$$\phi_{11}^{(0)} = \frac{2L_*^4 A^3 r_1}{a^3 \mathcal{A}_2^{3/2}} \cos(k_\perp \cdot x_\perp) [\partial_{\tau_+} g(\tau_+, \tau_-) + \partial_{\tau_-} g(\tau_+, \tau_-) + \mathcal{O}(m^2 a^2)]. \quad (71)$$

Adding up both contributions, we get that:

$$\phi_1 = -\frac{2L_*^4 A^3}{a^2 \mathcal{A}_2^{3/2}} g(\bar{\tau}_+, \bar{\tau}_-). \quad (72)$$

From this we get:

$$\frac{\phi_1}{\phi_0} = -\frac{2L_*^4 A^2}{a^2 \mathcal{A}_2} \left[\frac{g(\bar{\tau}_+, \bar{\tau}_-)}{f(\bar{\tau}_+, \bar{\tau}_-)} + \mathcal{O}\left(\frac{r_1^2}{a^2}\right) + \mathcal{O}(m^2 a^2) \right]. \quad (73)$$

Thus, we conclude that we need to consider the $\mathcal{O}(m^2 a^2 \frac{r_1}{a})$ corrections in order to see effects on r_* .

The only order $(m^2 a^2 \frac{r_1}{a})$ correction to ϕ_0 will be to φ_{01} since our expression for φ_{00} satisfies its free field equation of motion exactly. In order for (62) to be satisfied, we must have:

$$\begin{aligned} \varphi_{01}^{(1)} &= \frac{Ar_1 m^2 a}{4} [(\tau_- - \tau_0) \int_{\tau_0}^{\tau_+} d\tau'_+ f'(\tau'_+) \\ &\quad + (\tau_+ - \tau_0) \int_{\tau_0}^{\tau_-} d\tau'_- f'(\tau'_-)] \\ &= \frac{Ar_1 m^2 a}{4} [(\tau_- - \tau_0)(f(\tau_+) - f(\tau_0)) \\ &\quad + (\tau_+ - \tau_0)(f(\tau_-) - f(\tau_0))] \\ &\equiv \frac{Ar_1 m^2 a}{4} w(\tau_+, \tau_-), \end{aligned} \quad (74)$$

where τ_0 is an IR cutoff introduced to ensure that the massive wave modes do not spread out too far before coming together. We have also defined the function w for later convenience. Our expression for ϕ_0 is now:

$$\begin{aligned} \phi_0 &= \frac{A}{\sqrt{\mathcal{A}_2}} [f(\bar{\tau}_+, \bar{\tau}_-) - \frac{r_1 m^2 a}{4} \cos(k_\perp \cdot x_\perp) w(\tau_+, \tau_-) \\ &\quad + \mathcal{O}\left(\frac{r_1^2}{a^2}\right) + \mathcal{O}(m^4 a^4)]. \end{aligned} \quad (75)$$

We now need to consider the order $m^2 a^2 (\frac{r_1}{a})$ corrections to ϕ_1 . There are actually only three such contributions. One will come from the fact that our solution for ϕ_{11} (71) to the differential equation (70) neglected terms of order $m^2 a^2 (\frac{r_1}{a})$ (we will denote this contribution $\phi_{11}^{(1b)}$). Another contribution will be found by considering the RHS of (66) to linear order in $\varphi_{01}^{(1)}$ (we will denote this contribution $\phi_{11}^{(1a)}$). The last contribution will come from that fact we now need to consider a term of order $m^2 a^2 (\frac{r_1}{a})$ that comes from the linear contribution of $\varphi_{01}^{(0)}$ in the RHS of (66) (we will denote this contribution $\phi_{11}^{(1c)}$). All other corrections are of order $\frac{r_1^2}{a^2}$ or $m^4 a^4$. In order to get

$\phi_{11}^{(1b)}$ all we have to do is note that if we make the ansatz:

$$\begin{aligned}\phi_{11}^{(0)} &= \frac{2L_*^4 A^3 r_1}{a^3 \mathcal{A}_2^{3/2}} \cos(k_\perp \cdot x_\perp) \\ &\quad \int_{-\infty}^{\tau_+} d\tau'_+ \int_{-\infty}^{\tau_-} d\tau'_- h_b(\tau'_+, \tau'_-) \\ &\quad - \frac{L_*^4 A^3 r_1 m^2}{2a \mathcal{A}_2^{3/2}} \cos(k_\perp \cdot x_\perp) \\ &\quad \int_{\tau_0}^{\tau_+} d\tau'_+ \int_{-\infty}^{\tau'_+} d\tau''_+ \int_{\tau_0}^{\tau_-} d\tau'_- \int_{-\infty}^{\tau'_-} d\tau''_- h_b(\tau''_+, \tau''_-) \\ &\quad + \mathcal{O}\left(\frac{r_1^2}{a^2}\right) + \mathcal{O}(m^4 a^4),\end{aligned}\quad (76)$$

where

$$h_b(\tau_+, \tau_-) = \partial_{\tau_+} b(\tau_+, \tau_-) + \partial_{\tau_-} b(\tau_+, \tau_-), \quad (77)$$

then (70) is satisfied to order $m^2 a^2(\frac{r_1}{a})$. Thus:

$$\begin{aligned}\phi_{11}^{(1b)} &= -\frac{L_*^4 A^3 r_1 m^2}{2a \mathcal{A}_2^{3/2}} \cos(k_\perp \cdot x_\perp) \\ &\quad \int_{\tau_0}^{\tau_+} d\tau'_+ \int_{-\infty}^{\tau'_+} d\tau''_+ \int_{\tau_0}^{\tau_-} d\tau'_- \int_{-\infty}^{\tau'_-} d\tau''_- h_b(\tau''_+, \tau''_-).\end{aligned}\quad (78)$$

In order to find $\phi_{11}^{(1a)}$ we need to solve (66) with linear order contributions from $\varphi_{01}^{(1)}$ on the RHS. In this case, the equation becomes:

$$\begin{aligned}\square_4 \phi_{11}^{(1a)} &= \frac{2L_*^4 A^3 r_1 m^2}{a^3 \mathcal{A}_2^{3/2}} \cos(k_\perp \cdot x_\perp) [(\tau_+ - \tau_0) f'(\tau_+)^2 f''(\tau_-) \\ &\quad + (\tau_- - \tau_0) f'(\tau_-)^2 f''(\tau_+) \\ &\quad + 2f'(\tau_-) f''(\tau_+) ((\tau_+ - \tau_0) f'(\tau_-) + f(\tau_+) - f(\tau_0)) \\ &\quad + 2f'(\tau_+) f''(\tau_-) ((\tau_- - \tau_0) f'(\tau_+) + f(\tau_-) - f(\tau_0))] \\ &\equiv \frac{2L_*^4 A^3 r_1 m^2}{a^3 \mathcal{A}_2^{3/2}} \cos(k_\perp \cdot x_\perp) h_a(\tau_+, \tau_-).\end{aligned}\quad (79)$$

The solution up to the order we seek is:

$$\begin{aligned}\phi_{11}^{(1a)} &= -\frac{L_*^4 A^3 r_1 m^2}{2a \mathcal{A}_2^{3/2}} \cos(k_\perp \cdot x_\perp) \\ &\quad \int_{\tau_0}^{\tau_+} d\tau'_+ \int_{\tau_0}^{\tau_-} d\tau'_- h_a(\tau'_+, \tau'_-).\end{aligned}\quad (80)$$

The final correction of this order comes from the order $m^2 a^2(\frac{r_1}{a})$ term that arises from linear contributions from $\varphi_{01}^{(0)}$ on the RHS of (66):

$$\begin{aligned}\square_4 \phi_{11}^{(1c)} &= \frac{2L_*^4 A^3 r_1 m^2}{a^3 \mathcal{A}_2^{3/2}} \cos(k_\perp \cdot x_\perp) \\ &\quad [2(f'(\tau_+) + f'(\tau_-))] f'(\tau_+) f'(\tau_-) \\ &\equiv \frac{2L_*^4 A^3 r_1 m^2}{a^3 \mathcal{A}_2^{3/2}} \cos(k_\perp \cdot x_\perp) h_c(\tau_+, \tau_-).\end{aligned}\quad (81)$$

So we have that the order $m^2 a^2(\frac{r_1}{a})$ correction to ϕ_1 is:

$$\begin{aligned}\phi_{11}^{(1)} &= \phi_{11}^{(1a)} + \phi_{11}^{(1b)} + \phi_{11}^{(1c)} \\ &= -\frac{L_*^4 A^3 r_1 m^2}{2a \mathcal{A}_2^{3/2}} \cos(k_\perp \cdot x_\perp) \\ &\quad \left[\int_{\tau_0}^{\tau_+} d\tau'_+ \int_{-\infty}^{\tau'_+} d\tau''_+ \int_{\tau_0}^{\tau_-} d\tau'_- \int_{-\infty}^{\tau'_-} d\tau''_- h_b(\tau''_+, \tau''_-) \right. \\ &\quad \left. + \int_{\tau_0}^{\tau_+} d\tau'_+ \int_{\tau_0}^{\tau_-} d\tau'_- (h_a(\tau'_+, \tau'_-) + h_c(\tau'_+, \tau'_-)) \right] \\ &\equiv -\frac{L_*^4 A^3 r_1 m^2}{2a \mathcal{A}_2^{3/2}} \cos(k_\perp \cdot x_\perp) q(\tau_+, \tau_-).\end{aligned}\quad (82)$$

So, after adding in this contribution, we have:

$$\begin{aligned}\phi_1 &= -\frac{2L_*^4 A^3}{a^2 \mathcal{A}_2^{3/2}} g(\bar{\tau}_+, \bar{\tau}_-) [1 + \frac{m^2 r_1 a}{4} \cos(k_\perp \cdot x_\perp) \frac{q(\tau_+, \tau_-)}{g(\bar{\tau}_+, \bar{\tau}_-)} \\ &\quad + \mathcal{O}\left(\frac{r_1^2}{a^2}\right) + \mathcal{O}(m^4 a^4)].\end{aligned}\quad (83)$$

We thus get that:

$$\begin{aligned}\frac{\phi_1}{\phi_0} &= -\frac{2L_*^4 A^2}{a^2 \mathcal{A}_2} \frac{g(\bar{\tau}_+, \bar{\tau}_-)}{f(\bar{\tau}_+, \bar{\tau}_-)} \\ &\quad [1 + \frac{m^2 r_1 a}{4} \cos(k_\perp \cdot x_\perp) \left(-\frac{w(\tau_+, \tau_-)}{f(\bar{\tau}_+, \bar{\tau}_-)} + \frac{q(\tau_+, \tau_-)}{g(\bar{\tau}_+, \bar{\tau}_-)} \right) \\ &\quad + \mathcal{O}\left(\frac{r_1^2}{a^2}\right) + \mathcal{O}(m^4 a^4)].\end{aligned}\quad (84)$$

What we are interested in is the sign of the quantity: $(-\frac{w(\tau_+, \tau_-)}{f(\bar{\tau}_+, \bar{\tau}_-)} + \frac{q(\tau_+, \tau_-)}{g(\bar{\tau}_+, \bar{\tau}_-)})$. It is clear that $f(\bar{\tau}_+, \bar{\tau}_-)$ is always positive for $f(t) = e^{-t^2}$. Writing $g(\bar{\tau}_+, \bar{\tau}_-)$ explicitly for this ansatz for $f(t)$, we have:

$$\begin{aligned}g(\bar{\tau}_+, \bar{\tau}_-) &= \frac{1}{2} e^{2(\bar{\tau}_+^2 + \bar{\tau}_-^2)} [4e^{\bar{\tau}_-^2} \bar{\tau}_- \bar{\tau}_+ + 4e^{\bar{\tau}_+^2} \bar{\tau}_- \bar{\tau}_+] \\ &\quad + \sqrt{2\pi} e^{2\bar{\tau}_+^2 + \bar{\tau}_-^2} \bar{\tau}_+ (-2 + \text{erfc}(\sqrt{2}\bar{\tau}_-)) \\ &\quad + \sqrt{2\pi} e^{2\bar{\tau}_+^2 + \bar{\tau}_-^2} \bar{\tau}_- (-2 + \text{erfc}(\sqrt{2}\bar{\tau}_+)).\end{aligned}\quad (85)$$

It can be seen that $g(\bar{\tau}_+, \bar{\tau}_-) > 0$ for $\bar{\tau}_+, \bar{\tau}_- < 0$, which is the region we are interested in. Since:

$$\begin{aligned}&-\frac{w(\tau_+, \tau_-)}{f(\bar{\tau}_+, \bar{\tau}_-)} + \frac{q(\tau_+, \tau_-)}{g(\bar{\tau}_+, \bar{\tau}_-)} \\ &= \frac{1}{f(\bar{\tau}_+, \bar{\tau}_-) g(\bar{\tau}_+, \bar{\tau}_-)} [-w(\tau_+, \tau_-) g(\tau_+, \tau_-) \\ &\quad + q(\tau_+, \tau_-) f(\tau_+, \tau_-) + \mathcal{O}\left(\frac{r_1}{a}\right)].\end{aligned}\quad (86)$$

We are interested in the sign of the quantity:

$$\sigma(\tau_+, \tau_-) = -w(\tau_+, \tau_-) g(\tau_+, \tau_-) + q(\tau_+, \tau_-) f(\tau_+, \tau_-). \quad (87)$$

Let us plot $\sigma(\tau_+ = v, 0)$ in order to find the sign. We see in figure 4 that $\sigma(\tau_+ = v, 0)$ remains positive until

$v >> -1$. As we saw previously, the interaction length for the perfect planar case is of order a (which corresponds to the region where $v \sim -1$), so we see that increasing r_1 tends to increase r_* in the region where $r_1 \cos(k_\perp \cdot x_\perp)$ is positive. This is what we expected.

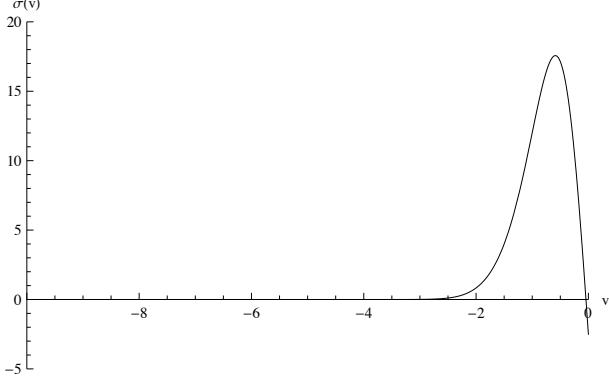


FIG. 4: $\sigma(\tau_+ = v, 0)$ as a function of v . $\tau_0 = -10$ was chosen, but any τ_0 much smaller than the τ_- , τ_+ where the interaction takes place defines an appropriate boundary condition.

VI. UV SENSITIVITY

We have so far explored a model with the Lagrangian:

$$\mathcal{L} = X + L_*^4 X^2, \quad (88)$$

where $X = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi$ is the standard kinetic term. Let us generalize this so as to include the possibility of a higher order non-linear term:

$$\mathcal{L} = X + L_*^{4n-4} X^n. \quad (89)$$

For the planarly symmetric case, we have for the first order EOM:

$$\square_2 \varphi_1 = -n L_*^{4n-4} \partial^\mu [\varphi_0 \partial_\mu (X_0^{n-1})], \quad (90)$$

where $X_0 = -\frac{1}{2A_2} \partial^\mu \varphi_0 \partial_\mu \varphi_0$. If we choose:

$$\varphi_0 = A f(\tau_+) + A f(\tau_-). \quad (91)$$

Then:

$$X_0 = \frac{2A^2}{A_2 l^2} f'(\tau_+) f'(\tau_-). \quad (92)$$

In this case, (90) simplifies to:

$$\begin{aligned} \square_2 \varphi_1 = & n(n-1) 2^n A^{2n-1} A_2^{-n+1} L_*^{4n-4} l^{-2n} \\ & [f'(\tau_+)^n f'(\tau_-)^{n-2} f''(\tau_-) \\ & + f'(\tau_-)^n f'(\tau_+)^{n-2} f''(\tau_+)]. \end{aligned} \quad (93)$$

The retarded solution to the above equation is:

$$\begin{aligned} \varphi_1 = & -A \epsilon \int_{-\infty}^{\tau_+} d\tau'_+ \int_{-\infty}^{\tau_-} d\tau'_- [f'(\tau'_+)^n f'(\tau'_-)^{n-2} f''(\tau'_-) \\ & + f'(\tau'_-)^n f'(\tau'_+)^{n-2} f''(\tau'_+)]. \end{aligned} \quad (94)$$

where

$$\epsilon = n(n-1) 2^{n-2} A^{2n-2} A_2^{-n+1} L_*^{4n-4} l^{2-2n}. \quad (95)$$

It can be seen by comparing figure 2 ($\varphi_1/(-A\epsilon)$ for the $n = 2$ case) to figure 5 ($\varphi_1/(-A\epsilon)$ for the $n = 3$ case) that the overall profile of φ_1 for odd and even values of n can be quite different. It should also be noted that we find relatively modest dependence of the profiles of two even (or two odd) φ_1 on n .

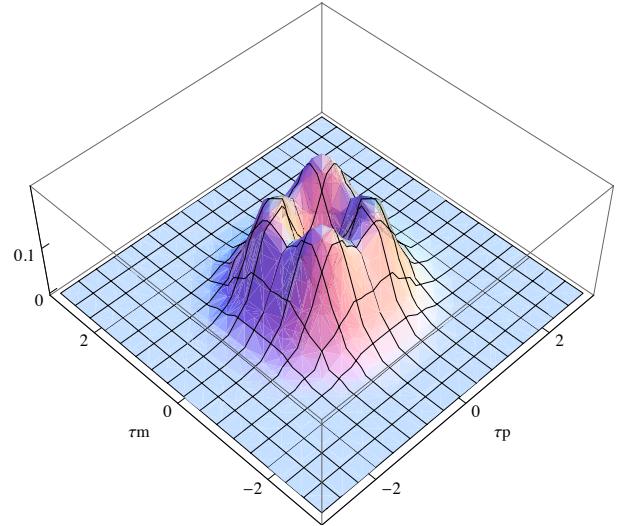


FIG. 5: The plot of $\varphi_1/(-A\epsilon)$ as a function of $\tau_\pm \equiv t_\pm/a$ for $f(\tau) = e^{-\tau^2}$, $n = 3$.

In order to compare the behavior of φ_1 for various values of n , let us plot the quantity:

$$\xi(v) = \frac{\varphi_1(\tau_+ = v, \tau_- = 0)}{\text{Max}_v[\varphi_1(\tau_+ = v, \tau_- = 0)]}. \quad (96)$$

The function $\xi(v)$ is the normalized peak amplitude of the right moving plane wave $f(\tau_-)$. We normalize by $\text{Max}_v[\varphi_1(\tau_+ = v, \tau_- = 0)]$, the maximum value of $\varphi_1(\tau_+ = v, \tau_- = 0)$ in the region $-\infty < v \leq 0$, so that the peak value of $\xi(v)$ is 1 regardless of what n we choose. Note that since we set $\tau_- = 0$, $v = \frac{2t}{a} = \frac{2x}{a}$. Thus, the right moving plane wave evolves from $v = -\infty$ to $v = 0$, where it will meet the left moving plane wave.

We see in figure 6 that $\xi(v)$ reaches its peak value at the value of $v = \frac{\sqrt{2}}{2}$ for all plots shown. Thus, the interaction length is $\sim a$ regardless of what value we choose for n . It is also apparent that as we increase n , there is a tendency for $\xi(v)$ to become more sharply peaked, so

ϕ_1 will become significant at somewhat smaller values of x as we increase n . Hence we can conclude that increasing n will not cause classicalization to occur for planarly symmetric waves. Moreover, the shape of the scattered wave is UV sensitive.

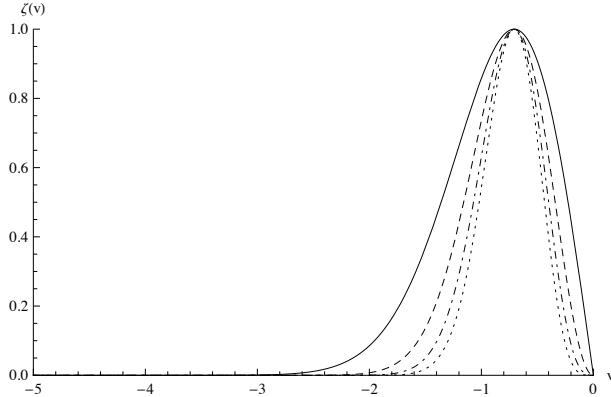


FIG. 6: The plot of $\xi(v)$ for $n = 2$ (solid line), $n = 3$ (dashed line), $n = 4$ (dot-dashed line), $n = 5$ (dotted line).

VII. CONCLUSION

We have pointed out that a field theory that exhibits the classicalization phenomenon for perfect spherical

symmetry ceases to do so when the spherical symmetry is significantly relaxed. We have shown that the classicalization radius tends to decrease in a region where a shell made of the field is flattened and that in the planar limit, the system never classicalizes before reaching sub-cutoff lengths. We have also seen that the shape of the scattered wave is UV sensitive. These considerations point towards the conclusion that classicalization does not serve as UV-completion for the class of non-renormalizable theories.

Acknowledgments

The work of RA and RS was supported in part by the US Department of Energy. Part of this work was done during S.M.'s visit at MCTP, University of Michigan, Ann Arbor. He would like to express his gratitude to the center for warm hospitality. S.M. acknowledges the support by Grant-in-Aid for Scientific Research 17740134, 19GS0219, 21111006, 21540278, by Japan-Russia Research Cooperative Program, and by WPI Initiative, MEXT, Japan.

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